STEADY UPWARD FLOW FROM WATER TABLES

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A. Anat, H. R. Duke and A. T. Corey

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NOTATIONS AND DEFINITIONS

Symbol	Term	Definition	Dimensions
	Capillary barrier	a sheet, strip, cup, or plate of porous material having finer pores than the soil medium in which it is in contact. Such a material will remain fully saturated with liquid and will prevent the entry of air into manometer leads, etc. after the soil has been at least partially desaturated. It will, however, permit the conduction of liquid through it.	
C	Conductivity	the coefficient in Darcy's equation when the medium is fully saturated. This coefficient is a function of both media and fluid properties. It is sometimes called "hydraulic conductivity."	Lt^{-1}
C_e	Effective conductivity	the coefficient in Darcy's equation when the medium is not necessarily fully saturated. It is sometimes called "capillary conductivity."	Lt^{-1}
C_e/C	Relative conductivity	a scaled conducitivity.	none
d	Distance	from a water table to the bottom of a dried surface layer of soil.	L
d.	Scaled distance	from a water table to the bottom of a dried surface layer of soil. It is the ratio $\frac{d}{p_b/\rho g}$. It could be called a dimensionless depth.	none
	Drainage cycle	a process in which the liquid pressure continuously decreases and the liquid saturation of the medium also decreases.	
	Evaporativity	the capacity of the atmospheric environment to evaporate water from an exposed free-water surface.	Lt^{-1}
	Hysteresis	As used here, the term refers to an unsteady flow process in which a decreasing liquid pressure changes to an increasing liquid pressure at some point in a porous medium. The process is often accompanied by little or no change in medium saturation.	
η		A property of the medium which can be related to the pore-size distribution and is defined in terms of the functional relation between C_e and p_c; i.e., $\eta = -\frac{\Delta(\ln C_e)}{\Delta(\ln p_c)}$ when $p_c > p_b$. Large values of η indicate a relatively uniform pore size and smaller values indicate a greater range of pore sizes.	
p	"Apparent" pressure	of liquid in soils. When referred to atmospheric pressure as a datum, it is the quantity measured by a tensiometer. It may sometimes differ from the hydrodynamic concept of pressure because of the adsorptive force fields acting near solid surfaces.	FL^{-2}
p_c	Capillary pressure	the pressure of the gas phase in soils minus the "apparent" pressure of the liquid. It is often called "suction" or "matric suction." When the gas pressure is atmospheric, $p_c = -p$. More generally, $p_c = p_a - p$ where p_a is the pressure of the air.	FL^{-2}
p_b	Bubbling pressure	approximately the p_c at which a soil begins to desaturate rapidly or at which the gas phase becomes continuous. A more rigorous definition is implicit in the method of its determination as described by Brooks and Corey [4].	FL^{-2}
p.	Scaled capillary pressure	the dimensionless ratio $\frac{p_c}{p_b}$.	none
q	Volume flux	the volume of flow per unit time per unit bulk area of medium.	Lt^{-1}

STEADY UPWARD FLOW FROM WATER TABLES[1/]

by

A. Anat, H. R. Duke, and A. T. Corey[2/]

INTRODUCTION

Upward flow from water tables subsequent to evaporation and transpiration from soils is a significant phenomenon, particularly in irrigated areas. In some areas, farmers find it advantageous to cause the water table to rise to within a few feet of the root zone of crops as a method of supplying the root zone with water. In areas where soluble salts may be substantial, however, such practices have often led to an unfavorable accumulation of salts within the root zone.

The hydrologist, also, is concerned with upward flow from water tables since such flow can have a substantial effect on the fluctuation of ground water storage.

Many investigators have studied rates of evapotion from soils in which there is a water table. There is general agreement that the rate of evaporation may be controlled by either the capacity of the atmospheric environment to evaporate water or the capacity of the soil to transmit water to the surface. The maximum rate of upward flow will be the lesser of these two capacities. Another conclusion that can be made from the available literature is that, except for very shallow water-table depths, the capacity of the soil to transmit water is the limiting factor. This paper is concerned with the latter situation.

Since hysteresis in the pressure history of soil-liquid systems plays an important role with respect to upward flow rates [12, 15, 16], this study investigates three cases as follows:

Case 1 - The soil liquid follows a drainage cycle only; i.e., the soil is initially completely saturated and afterwards the pressure of the liquid progressively decreases at all points in the system until steady state is reached. This case occurs in nature only when the maximum rate of evaporation from the surface never exceeds the rate at which water is supplied from the water table to the surface and when the water table is either at a constant elevation or is progressively lowered.

Case 2 - The soil liquid follows an imbibition cycle; i.e., the soil is initially dry and liquid is imbibed from below only.

Case 3 - The soil liquid follows a cycle which is neither completely drainage nor completely imbibition; i.e., there are parts of the soil profile in which the liquid pressure may at times increase and other parts in which the pressure may decrease continuously. Such cases are probably common in nature, especially when the rate of evaporation from the surface temporarily exceeds the upward flow rate.

This study, therefore, attempts to answer the following questions:

1. What is the maximum rate of upward flow (as a function of the depth at which liquid exists at atmospheric pressure) and what are the soil parameters that determine this maximum rate?

2. What is the maximum rate of upward flow when the liquid system follows an imbibition cycle completely?

3. How is the maximum rate of upward flow affected by a situation in which the removal of liquid from the surface exceeds the rate at which it can be transmitted to the surface?

1/ Contribution from the Agricultural Engineering Department, Colorado State University.

2/ AID Participant (SEATO Graduate School of Engineering), Junior Civil Engineer and Professor of Agricultural Engineering, Colorado State University, Fort Collins, Colorado, respectively.

PREVIOUS INVESTIGATIONS AT COLORADO STATE UNIVERSITY

In 1957 Staley [18] submitted a Masters Thesis which attempted to determine (1) how the evaporation rate from a fine sand is affected by independently varying the wind velocity and the depth of the water table, and (2) how the functional relationship between evaporation rates under specified ambient conditions and the depth of the water table can be related to easily measurable properties of the sand.

1. Wind-Tunnel Experiments

Two columns of a fine sand were placed side by side in a test section of a wind tunnel. Each column was 46 inches in length and 12 inches in diameter. The columns were placed so that their surfaces were flush with the floor of the test section. One column, in which the water table was maintained at the surface, was used to evaluate the evaporativity while the water-table depth in the second column was varied.

Both columns were initially fully saturated before the water table was lowered in one of the columns. The columns were supplied by water from a constant head source such that the rate of intake could be measured. Runs were made with the wind velocity varying from 0 to 50 feet per second while the water table was varied independently between the surface and a depth of 30 inches. The runs were conducted so that the column with the variable water table was always on the drainage cycle. Each run was continued until steady state flow was approached.

A critical depth of water table for the fine sand was found at about 24 inches. With the water table above this depth, evaporation rates were of the same order of magnitude as from the free-water surface. When the water table was below 24 inches, the flow rate decreased rapidly with depth and at 30 inches the rate was too small to be measured accurately with the apparatus employed.

The critical water-table depth was related to properties of the capillary pressure - desaturation curve. It was found that the critical depth corresponded to the value of $p_c/\rho g$ at which the initially saturated sand began to desaturate rapidly with further increases in p_c. This value of p_c has been called bubbling pressure p_b by Brooks and Corey [4].

A. Theoretical analysis - In analysing the problem of upward flow from water tables, Staley expressed the flow rate, the capillary pressure and the elevation above the water table in terms of scaled variables as follows:

(1) flow rate--the ratio of the flow rate to the hydraulic conductivity q/C,

(2) capillary pressure--the ratio of capillary pressure to the bubbling pressure p_c/p_b,

(3) elevation--the ratio of elevation above the water table to the bubbling pressure head $\frac{z}{p_b/\rho g}$.

Staley also derived an equation for the steady upward flow rate q (identical to equation 1) but used an approach somewhat different from that employed by Gardner. Staley wrote Darcy's law for the case of one-dimensional flow in the vertical direction and re-arranged this equation to give an expression explicitly describing the rate of change of p_c with z as a function of q and C_e, i.e.,

$$\frac{d\left(\frac{p_c}{\rho g}\right)}{dz} = 1 + \frac{q}{C_e} \quad (3)$$

When this equation is solved for z, the result is equation (1).

Staley expressed the functional relationship $C_e(p_c)$ in two parts rather than as the continous function proposed by Gardner. Staley assumed that C_e is a constant for $p_c < p_b$, and for $p_c > p_b$, C_e is given by

$$C_e = C\left(\frac{p_b}{p_c}\right)^{\eta} \quad (4)$$

where C is the value of C_e when media are fully saturated.

By a proper choice of the constants a and b in equation (2), the relationship proposed by Gardner and that proposed by Staley may result in practically identical functions for C_e when the value of η is the same.

Staley computed the value of η for his sand using a method proposed by Burdine [5] and arrived at a value of 16. This was surprising in view of the observation of Gardner that η would not be larger than about 4. After reviewing the literature of the petroleum industry, Staley concluded that a value of η equal to 8 should be about average for sands and that the larger value of η for his sand was due to its extremely uniform pore size. Later measurements by Schleusener [15], Scott and Corey [17] and Brooks and Corey [4] (using steady-state methods of measurement) indicate that an η of 16 is not uncommon but is, in fact, close to an average for a tightly packed unconsolidated sand.

Staley solved equation (1) by a numerical method for $\eta = 8$ and a particular water-table depth to obtain a relationship between $\frac{p_c}{p_b}$ (at the surface) and $\frac{q}{C}$. The water-table depth assumed was such as to give a value of $\frac{z}{p_b/\rho g}$ at the surface of 2.5. His solution verified the conclusion

Recent investigations at Colorado State University were specifically designed to answer the questions posed in the introduction.

1. Flow Controlled by Siphons

The first of these investigations has been described by Duke [8] in a Masters Thesis submitted in 1965. Duke attempted (by entirely different methods from those employed by previous investigators) to determine how the steady flow rates from water tables can be predicted for any soil with any water-table depth. He assembled and presented data with the purpose of making them usable for engineering estimations applicable to any field situations.

The use of these data, however, depends on the accurate determination of hydraulic conductivities, bubbling pressures, and the values of η for the soils under investigation. The significance and determination of these soil parameters have been discussed in detail by Brooks and Corey [4] and by Brooks [3]. The η referred to is the dimensionless exponent appearing in equation (4) which Brooks and Corey have related to the pore-size distribution of porous media.

A. Solution of flow equation - Duke analysed his data in terms of dimensionless variables introduced by Staley, i.e., q/C, p_c/p_b, $\frac{z}{p_b/\rho g}$, and η. He designated these as $q.$, $P.$, $Z.$, and η respectively, following the precedent established by Miller and Miller [13] and Brooks and Corey [4].

Duke adopted the empirical equation for the functional relationship $C_{\bar{c}}(P.)$ introduced by Staley, i.e., equation (4). He, therefore, wrote equation (1) in the form

$$d. = \frac{1}{1+q._m} + \int_1^{\infty} \frac{dP.}{1+q._m\, P.^{\eta}} \qquad (5)$$

where $d.$ is the scaled elevation from the water table to the bottom of a layer of dried surface soil, which is the particular value of $Z.$ at the elevation of the bottom of the dried soil layer. The subscript m implies that $q._m$ is the maximum value of $q.$, occurring when $P. \longrightarrow \infty$ at the scaled elevation $d.$.

Duke devised a graphical solution for equation (5) involving the determination of the area under the curves shown in figure 1. He also devised a computer program which determined the integral in equation (5) very quickly for integral values of η from 2 through 20. This range of η includes all values observed for porous media to date. This program was written to continue the solution until the area under the next increment (i.e., $\Delta Z.$ of the next successive increment) was less than 1×10^{-6}. That this cutoff point reduced the "tail" (see figure 1) to a negligible amount was proven by comparing the computer solutions (at η values of 2 and 3) with closed solutions of equation (5), similar to those presented by Gardner [9]. These two solutions were consistently alike to 6 significant figures. Closed solutions for $\eta > 4$ are not practical due to the complexity of the solutions.

A copy of the FORTRAN program (for an IBM 1401 computer) used for this solution is included in Appendix A. The solutions were compiled into a nomograph relating $q._m$ to $d.$ for each value of η. The complete nomograph is presented in figure 2 and the computed values are given in table 2 in Appendix A.

Although most of Duke's theoretical data were obtained using the computer program, the graph shown in figure 1 serves to illustrate a significant principle. The first term on the right of equation (5) is a constant for a particular value of $q._m$. Consequently, the plot maintains a constant value of $1/(1+q._m)$ up to $P. = 1$, then drops more or less rapidly (depending on the value of η) as $P.$ increases toward infinity. The value of $1/(1+q._m P.^{\eta})$ rapidly approaches zero as $P.$ is increased. This means that the increment of $Z.$ over which $P.$ changes from a relatively small value to a very high value is not large. It is possible, therefore, to establish the functional relationship between $q._m$ and $d.$ (with only small error) by ignoring the mechanism of transport through the drier soil near the surface. This is particularly true for materials having a relatively high value of η.

This is not to say that the mechanism of transport through the drier surface soil has little effect on the evaporation rate. It means, instead, that whatever factors affect the rate of transport through the surface layer also affect the thickness of the dry surface layer and, therefore, affect the distance from the water table to the dried layer. Knowing the distance from the water table to the dried layer and the necessary soil parameters (also whether the soil liquid has followed a drainage or imbibition cycle), it is possible to compute the flow rate. For definitions of drainage and imbibition cycles, see the section on notations and definitions.

B. Experimental methods - Duke devised a novel method of measuring flow rates for columns in which the capillary pressure was maintained at a zero value at a fixed elevation near the bottom of the column. Rather than depending on evaporation from the surface to produce upward flow, the liquid was removed from the surface through a capillary barrier attached to a siphon. Flow rates were

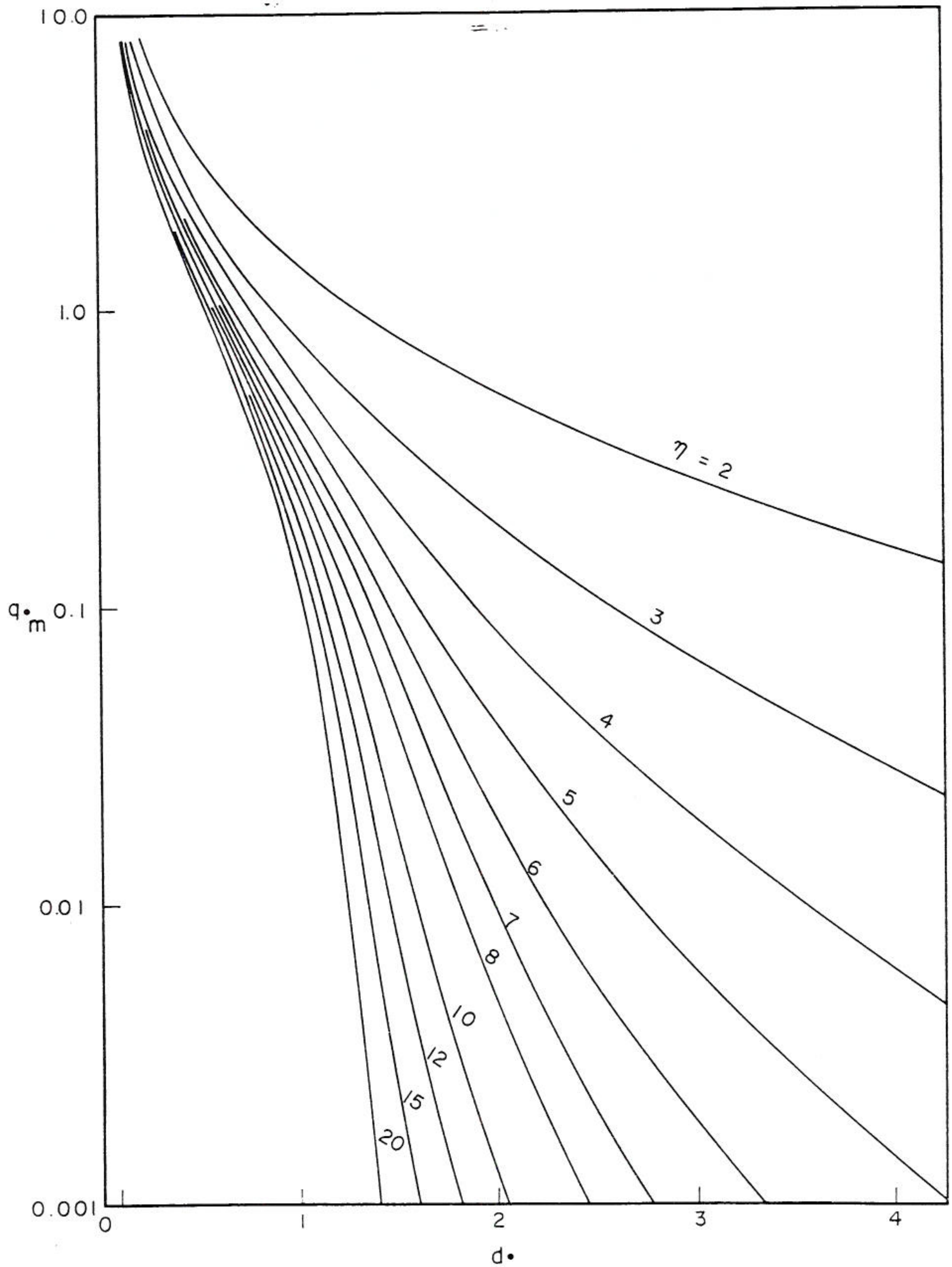

Figure 2. Nomograph of relation among $d_\bullet$, $q_{\dot{m}}$, and η.

FURTHER INVESTIGATIONS OF THE EFFECT OF HYSTERESIS

In June 1965, A. Anat completed an investigation [1] to determine more precisely the effect of hysteresis on upward flow from water tables. In his experiments, the equipment employed was the same as that used by Duke (see Appendix A). His procedures, in most cases, differed significantly from those followed by Duke.

1. Experimental Methods and Results

Anat packed his columns by first filling them with soil and then vibrating them with an electric powered vibrating device until they reached a relatively stable degree of compaction. The filling was accomplished by inserting a small tube (filled with soil and supplied by a funnel) and gradually withdrawing the tube as the column filled. This method avoided the necessity of dropping soil from the top of the column which might have resulted in segregation of particles according to size. It also avoided the segregation obtained by Duke from simultaneous filling and vibrating.

Using this packing technique, curves of relative conductivity as a function of capillary pressure were obtained which could be represented very well by equation (4). Both η and p_b were determined with precision as shown in figures 3 and 4.

A. Conductivity as a function of capillary pressure on the imbibition cycle - In some cases (see figure 4) curves of relative conductivity as a function of capillary pressure were also obtained on the imbibition cycle. This was accomplished by first draining the columns to a very low saturation (high p_c) and then raising the inflow reservoir and outflow siphon in increments. The conductivity was determined after steady state was obtained for each increment.

From these measurements, it was found that the conductivity at particular values of capillary pressure on the imbibition cycle is one or two orders of magnitude less than on the drainage cycle. The exception is for capillary pressures less than the bubbling pressure, in which case, the conductivity on the imbibition cycle is about half that on the drainage cycle, and the values of p_b were about 0.6 those obtained on the drainage cycle.

It has been pointed out by Bloomsburg and Corey [2] that for capillary pressures less than the bubbling pressure on the imbibition cycle, the conductivity is a function of time since the entrapped air will eventually diffuse from the system and the medium will become fully saturated.

B. Measurement of upward flow rates - The procedure used by Anat for determining maximum upward flow rates differed from that employed by Duke in several ways. The columns used were usually only slightly longer than necessary for determining conductivity as a function of capillary pressure. Instead of shortening the column after a series of runs, Anat simulated greater depths to a water table by lowering the inflow siphon. He was careful, however, not to desaturate the column below the lower tensiometer ring.

When the system reached a steady state, the lower tensiometer was read and the outflow rate was measured. Knowing the fully-saturated conductivity of the soil, it was possible to compute an equivalent depth to a zone of zero capillary pressure by using Darcy's law.

The first runs were made with the zone of zero capillary pressure at the elevation of the lower tensiometer. The capillary pressure at the surface was gradually increased in very small increments until the outflow appeared to approach a maximum, but the outflow siphon was not lowered more than this.

After this, the inflow siphon was lowered to simulate a greater depth to the water table, but the elevation of the outflow siphon was not changed. This procedure resulted in no pressure reversal at the upper tensiometer. Furthermore, the flow rates agreed very closely with those predicted from equation (5). This fact is shown in figure 5.

It will be noted that Anat plotted $q_{\dot{m}}$ as a function of d. on log-log paper. His reason for doing this was that, theoretically, this curve should approach a slope of η at low values of $q_{\dot{m}}$, thus indicating a relationship to the relative conductivity curves shown in figures 3 and 4. This result was experimentally confirmed as figure 5 shows.

C. Upward flow on the imbibition cycle - One series of runs was also made by starting with completely dry soil and allowing the soil to imbibe liquid from the lower inflow barrier. The initial imbibition was produced with the inflow siphon at a low elevation. The imbibition took place at an extremely slow rate and a considerable time passed before any flow from the outflow siphon was observed. Readings of the outflow rate were finally made, but it is extremely unlikely that sufficient time was allowed for the flow rate to reach a steady state (or a maximum rate for the particular elevation of the inflow siphon.)

When the inflow siphon was raised, the system reached steady state at increasingly shorter times, and it is probable that the data obtained represented maximum upward flow rates. At any rate, the data for the higher elevation of the inflow siphon agreed very closely with theoretical rates computed using conductivity data obtained on an imbibition cycle as figure 6 shows.

Another series of runs was made in which the imbibition cycle was started after draining the soil

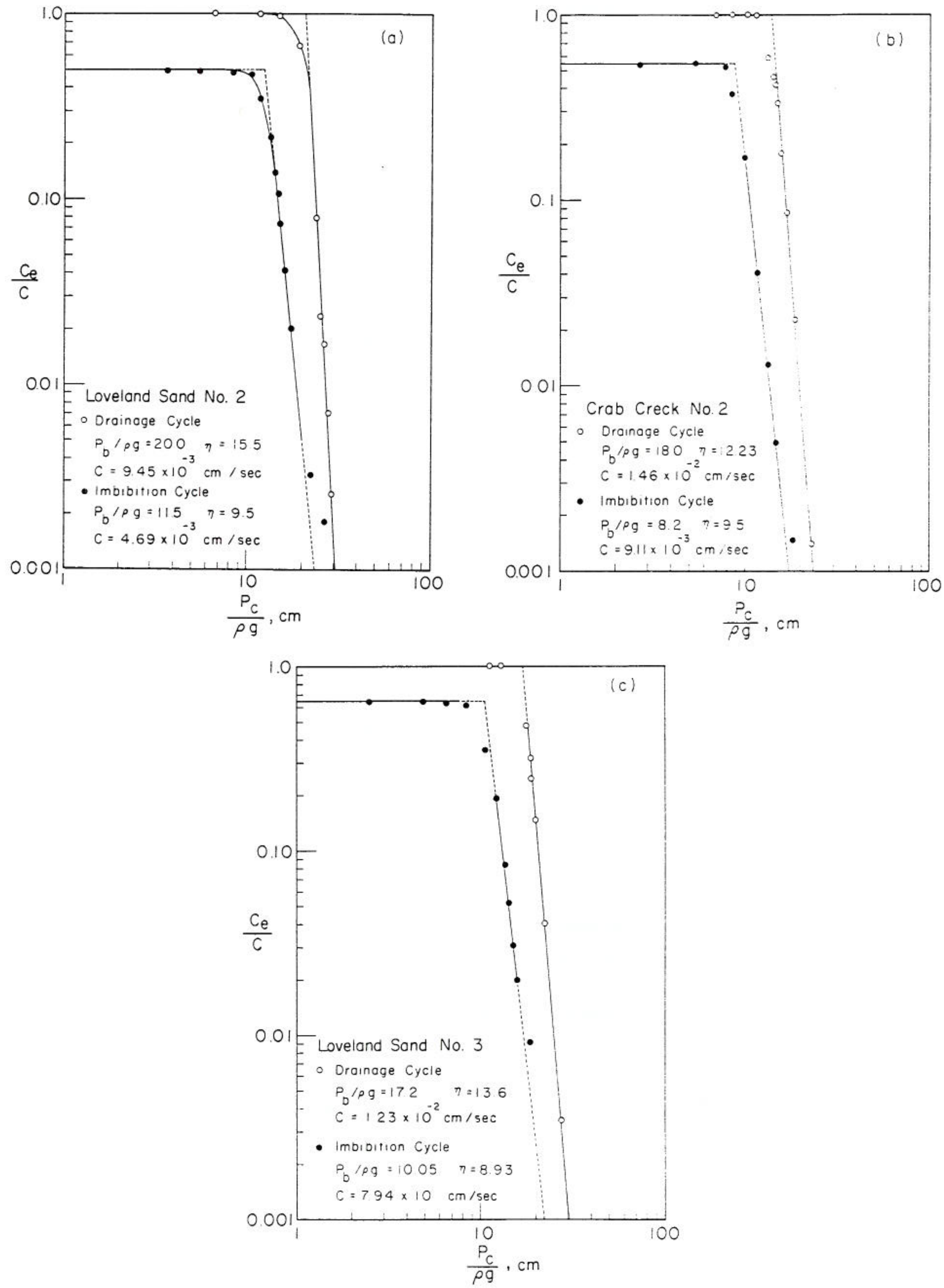

Figure 4. Relative conductivity - capillary pressure curves for drainage and imbibition cycles.

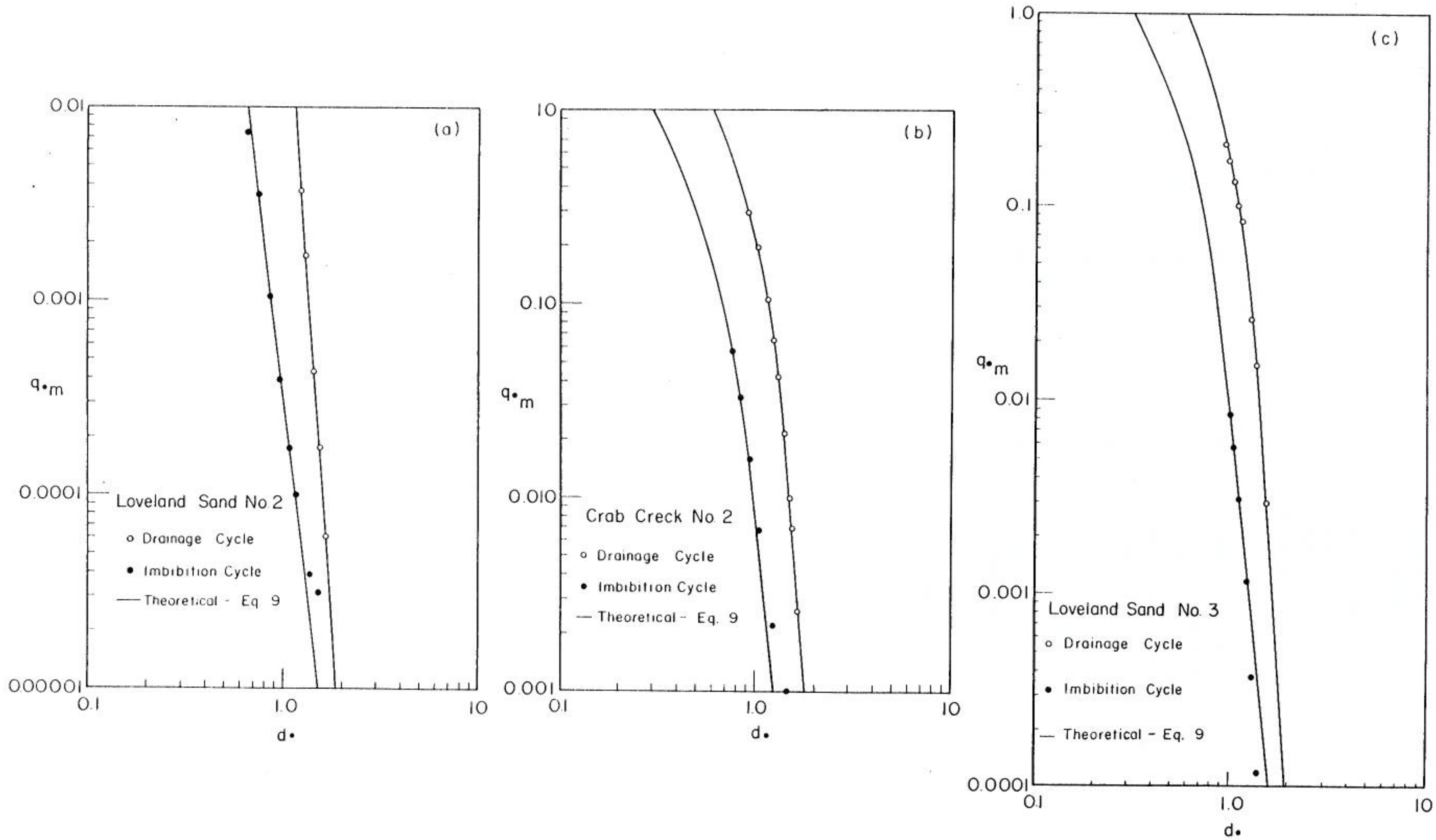

Figure 6. Comparison of experimental values of q_m as a function of d. with computed values for drainage and imbibition cycles.

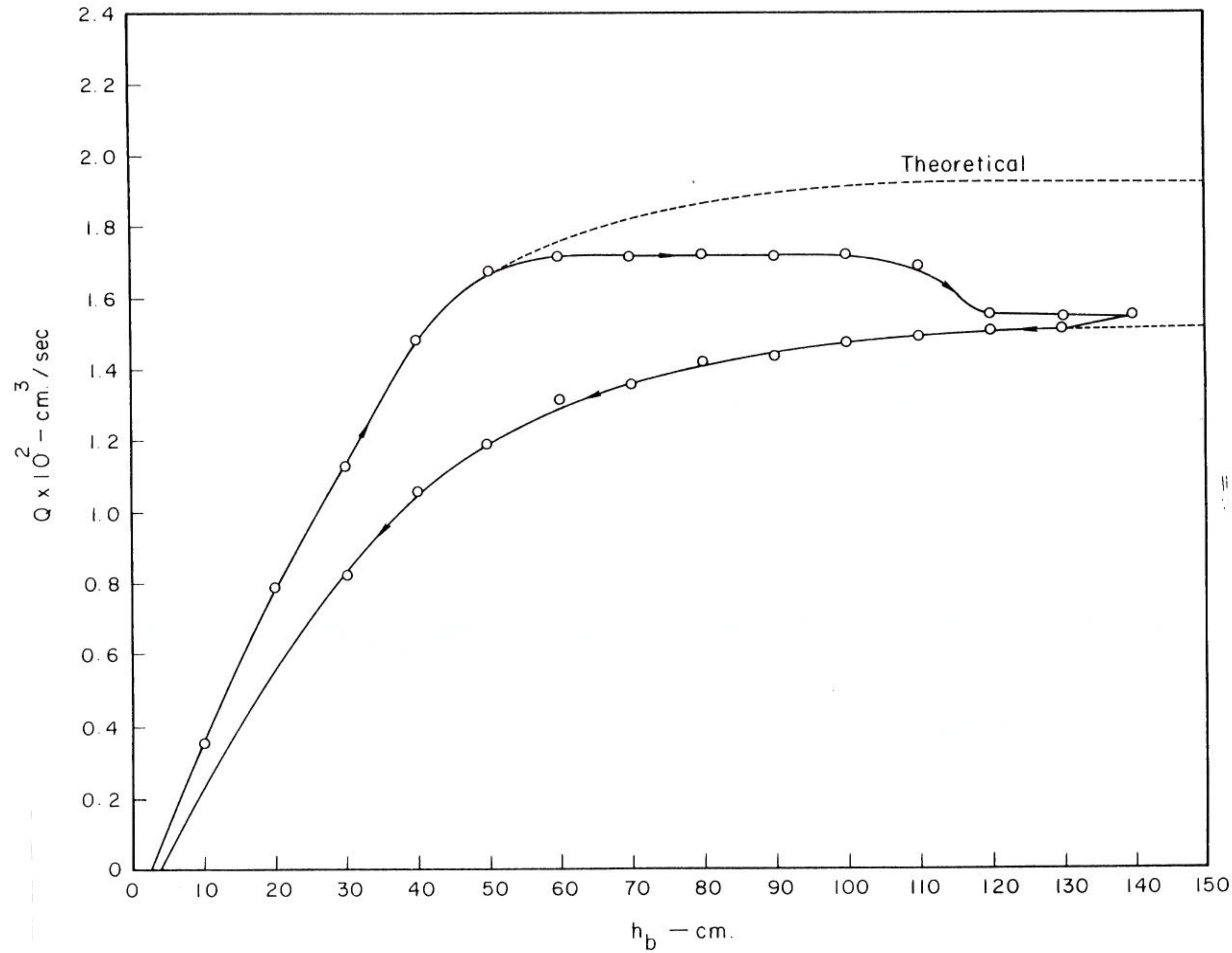

Figure 7. Effect of hysteresis on flow rate as a function of negative head at outflow barrier.

EQUATION FOR APPROXIMATING MAXIMUM UPWARD FLOW RATES

Anat also developed an algebraic equation which approximates very closely the values determined from equation (5) using a computer. His derivation is summarized as follows:

Equation (5) is first written as

$$d_{\cdot} = \frac{1}{1+q_{\dot{m}}} + \frac{1}{q_{\dot{m}}^{1/\eta}} \int_{q_{\dot{m}}^{1/\eta}}^{\infty} \frac{dx}{1+x^{\eta}} \qquad (6)$$

in which x is a new variable representing $P_{\cdot} q_{\dot{m}}^{1/\eta}$. To solve equation (6), the value of the integral

$$\int_0^x \frac{dx}{1+x^{\eta}}$$

must be known. The exact value of this integral when $\eta > 4$ is extremely complex. The technique employed is to expand $\frac{1}{1+x^{\eta}}$ into a convergent series and integrate it term by term.

The convergent series of $\frac{1}{1+x^{\eta}}$ is not the same when $x < 1$ as when $x > 1$. The values of

$$\int_0^x \frac{dx}{1+x^{\eta}} ,$$

therefore, are separated into two cases, i.e., $x < 1$ and $x > 1$.

By the foregoing techniques, approximate values of the integral are as follows:

Case 1 -- $x < 1$

$$\int_0^x \frac{dx}{1+x^{\eta}} \approx x - \frac{x}{\eta+1} \ln(1+x^{\eta}) + \frac{x^{2\eta}}{4(\eta+1)^2} . \qquad (7)$$

Case 2 -- $x > 1$

$$\int_0^x \frac{dx}{1+x^{\eta}} \approx 1 + \frac{1.886}{\eta^2+1} - \frac{x}{\eta-1} \ln(1+x^{-\eta}) - \frac{x^{-\eta}}{(2\eta-1)^2} . \qquad (8)$$

Substituting equations (7) and (8) into equation (6), the values of $d_{\cdot}$ corresponding to maximum values of $p_b/\rho g$, C, and η are as follows:

Case 1 -- $q_{\dot{m}} < 1$

$$d_{\cdot} \approx \frac{1}{q_{\dot{m}}^{1/\eta}} \left(1 + \frac{1.886}{\eta^2+1}\right) - \frac{q_{\dot{m}}}{1+q_{\dot{m}}} + \frac{\ln(1+q_{\dot{m}})}{\eta+1} . \qquad (9)$$

Case 2 -- $q_{\dot{m}} > 1$

$$d_{\cdot} \approx \frac{1}{1+q_{\dot{m}}} + \frac{1}{\eta-1} \ln\frac{(1+q_{\dot{m}})}{q_{\dot{m}}} . \qquad (10)$$

The subscript m in equations (9) and (10) indicates that $q_{\dot{m}}$ represents the maximum value of $q_{\cdot}$, obtained when $p_c \rightarrow \infty$ at the scaled elevation $d_{\cdot}$. The equations (9) and (10) are sufficiently accurate when $\eta \geq 4$. This is not, however, a serious disadvantage since the values of $I(x)$ can be obtained easily by direct integration when $\eta = 2$ or 3.

The degree of accuracy of equations (9) and (10) is shown by comparing the values computed from them with the values computed by Duke which are shown in table 2 in Appendix A. A comparison of values (for $\eta = 4$, 8, and 12) computed from the approximate equations with those obtained by Duke are shown in table 1 on the next page. A detailed discussion of the derivation is given in the Ph.D. Dissertation by Anat [1].

For shallow water-table depths or for large values of $q_{\cdot}$, the flow rate is controlled by external evaporative conditions. The authors suspect that equation (10) is of academic interest only. For most actual problems, the value of $q_{\dot{m}}$ is small and if the value of $q_{\dot{m}} < 0.01$, equation 9 can be approximated by

$$d_{\cdot} \approx \frac{1}{q_{\dot{m}}^{1/\eta}} \left(1 + \frac{1.886}{\eta^2+1}\right) \qquad (11)$$

or

$$q_{\dot{m}} \approx \left(1 + \frac{1.886}{\eta^2+1}\right)^{\eta} d_{\cdot}^{-\eta} . \qquad (12)$$

Equation (12) is somewhat comparable in form to those which appear in Gardner's paper [9] as equations (18, 19, 20, and 21). The latter, however, were for integral values of η from 1 to 4 only. In deriving these equations, Gardner also assumed that q was small so that q/a approached zero, a being a soil parameter.

An important advantage of equations (9, 10, 11, and 12) over the particular computer program employed by Duke is that the former can be solved for non-integral values of η. Furthermore, these equations will greatly facilitate computation of flow rates through stratified soil even though a computer may be necessary for such computations. The theory of how such computations might be made is presented in the Thesis by Anat [1]. It has not been verified experimentally to date.

SUMMARY AND CONCLUSIONS

Experiments were conducted at Colorado State University to investigate steady upward flow from water tables. The first investigations began in 1956 (as part of the Western Regional Project W-32) some aspects of which have been reported in previously published literature [12, 16]. The principal purpose of this paper is to report the findings of studies which were completed in 1965 concerning upward flow from water tables.

The objective of the latter studies was to answer the questions:

1. What is the maximum rate of upward flow (as a function of the depth at which liquid exists at atmospheric pressure) and what are the soil parameters that determine this maximum rate?

2. What is the maximum rate of upward flow when the liquid system follows an imbibition cycle completely?

3. How is the maximum rate of upward flow affected by a situation in which the removal of liquid from the surface exceeds the rate at which it can be transmitted to the surface?

A differential equation was developed which relates the maximum scaled flow rate $q_{\dot{m}}$ to the scaled distance $d.$ from the zone of zero capillary pressure to the bottom of a dry surface layer. The differential equation contains the soil parameter η which is a dimensionless number evaluating the pore-size distribution.

The flow rate q is scaled by dividing it by the hydraulic conductivity of the fully saturated soil and the distance d is scaled by dividing by the quantity $p_b/\rho g$, in which p_b is the capillary pressure at which the soil begins to desaturate rapidly.

The differential equation relating $q_{\dot{m}}$ to $d.$ has been solved using a computer program for a range of η from 2 to 20, which includes all values of η found to date. This program, however, is suitable for values of η that are integers only. Algebraic expressions were also developed which approximate the exact solution of the differential equation. The approximation is close enough for any conceivable practical purpose and, furthermore, the algebraic expressions can be evaluated for values of η that are non-integers as well as integers.

It was found that the theoretical equations gave functional relations between $q_{\dot{m}}$ and $d.$ which agreed extremely well with measured functional relations when the soil parameters C, p_b, and η were measured with sufficient accuracy. The agreement was good for systems in which the soil liquid followed both an imbibition (rising water table) and a drainage cycle (falling water table), provided the soil parameters were obtained for the appropriate cycle.

Unfortunately, however, the values of $q_{\dot{m}}$ corresponding to a particular value of $d.$, when the system follows a drainage cycle can be 100 times greater than when the system follows an imbibition cycle. Furthermore, whenever the rate of removal of liquid from a soil surface substantially exceeds the rate at which the liquid can be replaced from the interior of the soil, hysteresis takes place.

The hysteresis process probably takes place as follows:

A large potential gradient at the surface at first removes liquid from the surface layer faster than the liquid can be replaced from the interior. The desaturation of the surface layer results in a large reduction in its conductivity so that a stage is reached when the rate at which the desaturated layer can imbibe water from the interior is somewhat greater than the rate at which it can be conducted through the layer. This causes the pressure of water entering the desaturated layer to increase, thus decreasing the potential gradient producing flow from the interior by the time steady state flow is established. Because the functional relations among degree of saturation, pressure and conductivity contain hysteresis loops, the increased pressure is not accompanied by a material increase in saturation or conductivity in the surface layer.

The result is that when flow takes place from a water table to a soil surface (under conditions that would otherwise produce a drainage cycle) quick drying of the surface will produce flow rates less than when hysteresis does not occur.

The reduction in flow rate, furthermore, depends on the rate at which the surface layer is desaturated; the faster it is desaturated, the greater the reduction in flow rate. Reduction in flow rates of from 20 to 50 percent resulting from hysteresis have been observed. It is possible, of course, that if the surface layer had been desaturated even more quickly than was possible with the apparatus employed, reduction in flow rates greater than 50 percent might have occurred.

APPENDIX A

(From Thesis by Duke [8])

Fortran Computer Program for Solution of Equation (5)

SEQ	STMNT	FORTRAN STATEMENT
	C	PROGRAM FOR HAROLD DUKE
1		H=.05
2		PI=3.1415927
3		N=1
4		DO 20 LLL=1,19
5		N=N-1
6		EN=N
7		Q=.0005
8		DO 20 LL=1,14
9		ZZ=0.0
10		Q=Q*2
11		Z=PI/(EN*SINF(PI/EN)*Q**(1./EN))
12		DO 10 L=1,10
13		R=FLOATF (L)-1.
14		TERM=(H/3.)*((1./(1.+Q*((2.*R*H)**N))) +
		1 (4./(1.+Q*((2.*R*H+H)**N))) +
		(1./(1.+Q*((2.*R*H+H+H)**N))))
15	00010	ZZ=ZZ+TERM
16		Z=Z+ZZ
17		ZZZ=1./(1.+Q)
18		Z=Z+ZZZ
19		PUNCH 1020, N,Q,Z
20	00020	PRINT 1010,N,Q,Z
21		STOP
22	01020	FORMAT (3H N=12,5X,3H Q=P6.3, 5X, 3H Z=E18.8)
23		END

APPENDIX B

(From Thesis by A. Anat [1])

TABLE 3

RELATIVE PERMEABILITY-CAPILLARY PRESSURE DATA

Soil	$p_c/\rho g$(cm.)	C_e/C
Loveland Sand No. 1		
Drainage cycle	10.70	1.0000
	13.00	1.0000
$C = 1.047\text{x}10^{-2}$ cm/sec	15.45	0.8960
	15.80	0.8100
$p_b/\rho g$ = 18.00 cm	17.80	0.5020
	20.30	0.2000
η = 12.3	22.00	0.0874
	24.80	0.0226
	25.0	0.016
Touchet Silt Loam		
Drainage cycle	9.65	1.0000
	24.80	1.0000
$C = 2.85\text{x}10^{-4}$ cm/sec	37.05	1.0000
	43.50	0.9770
$p_b/\rho g$ = 72.2 cm	53.10	0.9090
	60.30	0.8600
η = 6.2	68.30	0.6800
	71.50	0.6600
	91.35	0.2330
	106.90	0.09680
	107.50	0.07550
	117.50	0.04870
	127.60	0.03200
Loveland Sand No. 2.		
Drainage cycle	6.60	1.00000
	11.30	1.00000
$C = 9.45\text{x}10^{-3}$ cm/sec	14.50	1.00000
	18.50	0.69200
$p_b/\rho g$ = 20.0 cm	23.50	0.10200
	25.10	0.00234
η = 15.2	26.10	0.00166
	27.80	0.00071
	29.30	0.000262
Imbibition cycle	26.20	0.00018
	22.25	0.00033
$C = 4.685\text{x}10^{-3}$ cm/sec	17.50	0.02050
	16.00	0.04150
$p_b/\rho g$ = 12.2 cm	15.00	0.07514
	14.60	0.10920
η = 9.5	13.20	0.21100
	11.80	0.34870
	10.30	0.46950
	8.20	0.47700
	5.40	0.48500
	3.60	0.48960

TABLE 4

SUMMARY OF MAXIMUM RATES OF UPWARD FLOW,

COMPARED WITH THEORETICAL VALUES (EQUATION 9)

Soil	$qx10^2$ cm/sec	$Cx10^2$ cm/sec	$q_{\dot{m}}$	Experimental d.	Eq. 9 d.
Loveland Sand No. 1					
Drainage cycle	0.80800	1.047	0.77200	0.647	0.639
	0.74060	"	0.70700	0.676	0.667
$p_b/\rho g$ = 18.00 cm	0.61300	"	0.58600	0.718	0.723
	0.47300	"	0.45200	0.795	0.797
η = 12.3	0.40900	"	0.39030	0.843	0.837
	0.31000	"	0.30200	0.920	0.904
	0.25200	"	0.24130	0.977	0.959
	0.20300	"	0.19400	2.034	1.008
	0.13500	"	0.12800	1.112	1.093
	0.06200	"	0.05930	1.246	1.225
	0.00870	"	0.00830	1.456	1.495
	0.00200	"	0.00190	1.772	1.710
Touchet Silt Loam					
Drainage cycle	0.02663	0.0285	0.93500	0.625	0.667
	0.02062	"	0.72400	0.763	0.759
$p_b/\rho g$ = 72.2 cm	0.01617	"	0.50800	0.874	0.888
	0.00857	0.0300	0.28400	1.120	1.098
η = 6.2	0.00291	"	0.09600	1.573	1.453
	0.00182	"	0.06100	1.650	1.596
Loveland Sand No. 2					
Drainage cycle	0.03610	0.943	0.03820	1.140	1.210
	0.02580	"	0.03660	1.223	1.219
$p_b/\rho g$ = 20.0 cm	0.01590	"	0.01690	1.313	1.300
	0.00400	"	0.00428	1.407	1.436
η = 15.2	0.00160	"	0.00173	1.528	1.526
	0.00058	0.950	0.00061	1.625	1.637
Imbibition cycle	0.00080	0.943	0.00008	1.220	1.156
	0.00094	"	0.00010	1.148	1.123
$p_b/\rho g$ = 11.5	0.00161	"	0.00017	1.045	1.065
	0.00370	"	0.00039	0.965	0.971
η = 9.5	0.00970	"	0.00104	0.865	0.869
	0.03340	"	0.00355	0.739	0.738
	0.06980	"	0.00743	0.650	0.648
Crab Creek No. 2					
Drainage cycle	0.40720	1.381	0.29500	0.919	0.912
	0.27130	"	0.19640	1.020	1.007
p_b = 13.50 cm	0.14640	"	0.10600	1.138	1.129
	0.09080	"	0.06520	1.216	1.211
	0.05900	1.393	0.04230	1.289	1.274
η = 12.2	0.03200	"	0.02297	1.377	1.360
	0.01400	1.381	0.01040	1.489	1.470
	0.00980	"	0.00709	1.542	1.513
	0.00365	"	0.00264	1.627	1.647
Imbibition cycle	0.00146	1.447	0.00103	1.443	1.270
	0.00318	"	0.00225	1.221	1.132
$p_b/\rho g$ = 8.2 cm	0.00993	"	0.00684	1.040	0.985
	0.02340	"	0.00162	0.925	0.923
η = 9.0	0.04870	"	0.00337	0.827	0.836
	0.08340	"	0.00569	0.754	0.770

EQUIPMENT AND PROCEDURES

Design of Equipment

The individual components used in Duke's research and subsequent studies of a similar nature were those designed and built by Brooks and Corey. Figures 9, 10, and 11 show sketches of the several components. Most of the length of the soil columns was enclosed by acrylic plastic cylinders as shown in figure 9(a). Each of these sections had an annular groove machined into the wall at the bottom end of the section with small notches connecting this groove to both inside and outside the tubing. The purpose of these grooves was to provide a means for air to enter the column as drainage proceeded and to minimize the escape of liquid by capillary flow through the joints.

Inflow to the column was supplied through a section of acrylic tubing closed on the lower end and fitted with a capillary barrier as shown in figure 10. This section was machined on the inside to allow a thin sleeve of Selas filter material with a bubbling pressure of 10 psi, having the same inside diameter as the column sections, to be inserted. An annular space outside the barrier allowed free passage of liquid around its outer surface. This space was connected to an inflow siphon by means of a 1/4" drilled and tapped hole and plastic tubing.

Fluid was removed from the top of the soil column through a ceramic barrier of the same composition as that used for the inflow sleeve. The outflow barrier consisted of a disc of ceramic cemented to a plastic section milled from solid stock, having a center bore and provision for attachment of tubing at the upper end. This barrier was of such size as to slip freely into the column sections to allow good contact between barrier and soil, yet prevent excessive evaporation. This arrangement is shown in figure 11.

To provide for measurement of capillary pressures within the columns, sections of cylinders 2 cm in length were machined inside to form an annular groove to retain thin rings of ceramic, 1/2 cm wide, the arrangement being shown in figure 9(b). The annular groove was connected to a small displacement manometer. These tensiometer rings operated in the same manner as ordinary tensiometers except that, being flush with the inside wall of the column sections, they did not reduce the cross-sectional area of the soil column. All capillary barriers including those used in the tensiometers were cemented to the acrylic plastic using Armstrong A-1 industrial adhesive.

For the lower flow rates, the outflow was measured in a 4 mm O.D. glass tube calibrated in cm^3 per cm of length. The higher flow rates were measured by allowing the liquid to drip from the end of the tube into a 5 ml burette. Figure 12 shows both the outflow measuring device and the constant head inflow reservoir.

For soil materials having a very high permeability and low bubbling pressure, Porvic plastic was substituted for Selas as barrier material to reduce the head loss through the barriers and to maintain better control of the capillary pressures.

Procedure

The columns were assembled by fastening together the acrylic sections mentioned previously. The inflow section was at the bottom of the column, with a tensiometer ring immediately above to provide a reference for the water table.

A 4 cm and a 2 cm cylinder section were placed above the lower tensiometer ring. A second tensiometer ring was placed above these two column sections. This portion of the column provided a convenient section for measuring conductivity as a function of capillary pressure. The latter measurements were made after the upward flow experiments were completed. The alternate placement of column sections and tensiometer rings until the desired length was attained completed the column arrangement for the upward flow tests.

The sections were then taped together and packed using a mechanical packer of a type devised by Jackson et.al.[11]. Experiments showed that this method resulted in a uniform bulk density throughout the column. Later experience has shown, however, that this packer (or any device which simultaneously vibrates and fills a soil column) causes large particles and aggregates to concentrate at the wall of the column thus producing non-homogeneity in horizontal planes. This circumstance adversely affected some of Duke's results.

After the columns were packed, a disc of fiberglass mat was placed over the top of the soil column to prevent erosion during the saturation procedure. A plastic plug containing holes to permit the escape of air held the fiberglass against the soil and retained the soil in the column. The column was then submerged in a container of hydrocarbon to which a vacuum was applied. The vacuum was maintained until air bubbles ceased to emerge from the soil and the capillary barriers. Afterwards the vacuum was removed allowing the liquid to fully saturate the soil and the capillary barriers.

The saturated columns were removed from the liquid container and placed in a horizontal position to prevent desaturation. Short sections of oil-filled tubing were connected to all manometer taps and clamped to prevent air entering the capillary barriers. The column was then cut even with the top of the upper tensiometer section to give a smooth soil surface and fastened to a vertical channel iron near the manometer board.

The inflow reservoir was immediately attached to the inflow barrier to prevent the zone of zero

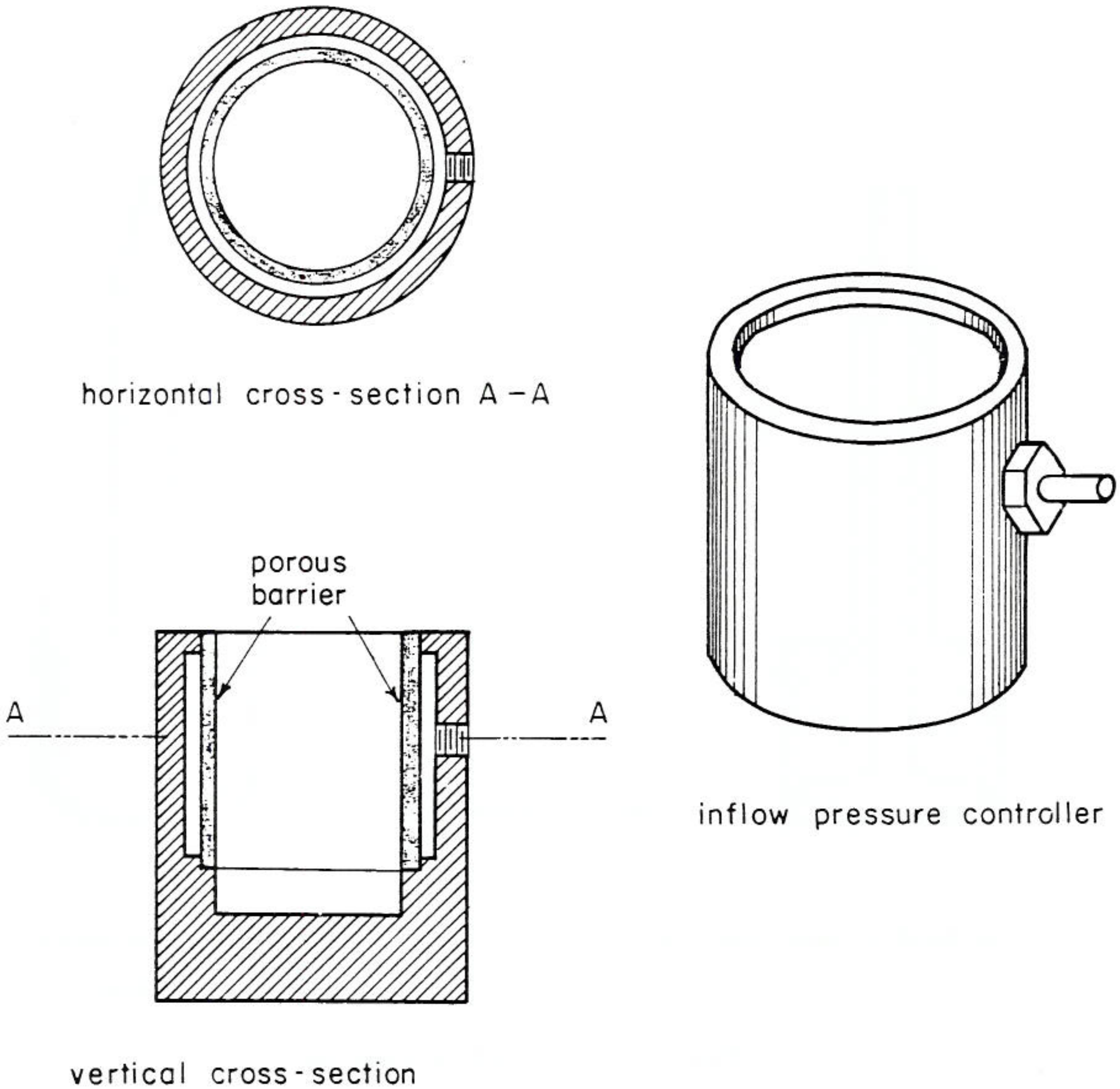

Figure 10. Inflow pressure controller.

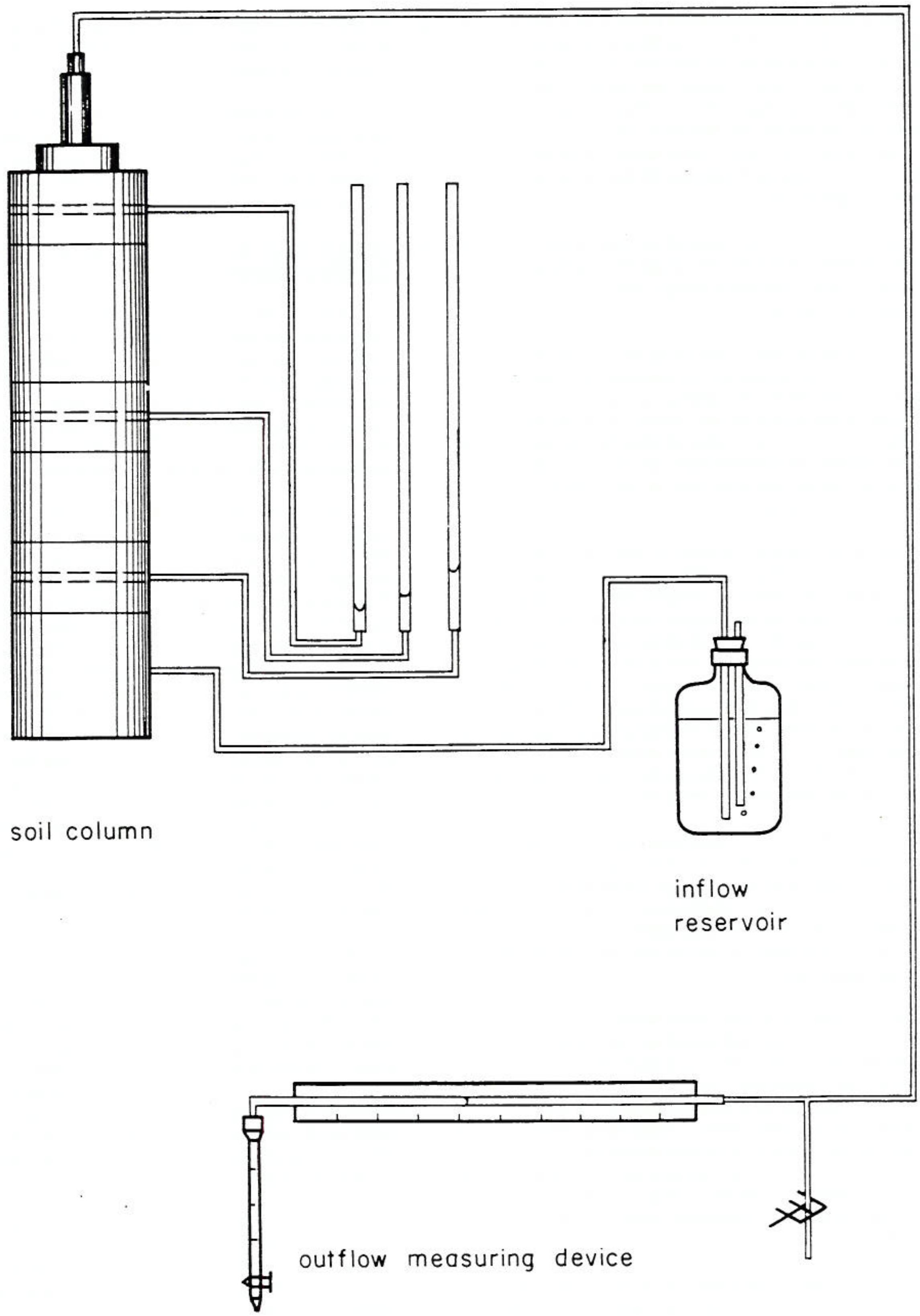

Figure 12. Schematic of apparatus for upward flow experiments.

Key Words: Evaporation, Upward Flow, Water Table, Porous Media, Soil

Abstract: Steady upward flow from water tables was investigated experimentally and theoretically under conditions in which the flow rate is controlled by the capacity of the soil to transmit liquid to a dry surface layer. Under these conditions the flow rate was found to depend upon whether the soil-liquid system was on a drainage (falling water table) or an imbibition (rising water table) cycle. For a given depth of water table, the flow rate on the drainage cycle can be at least 100 times greater than on the imbibition cycle.

It was also found that whenever the rate of removal of liquid from the surface layer exceeds the rate at which it can be supplied from the soil below, the flow rate is reduced substantially compared to the flow rate which would otherwise exist on a drainage cycle.

A differential equation was developed which relates the upward flow rate to the depth of the water table in terms of measureable soil parameters. A Fortran program for a IBM computer was developed to solve the differential equation for a wide range of soil parameters. An algebraic expression was also developed which approximates the computer solution very closely.

(Abstract continued on reverse side)

Reference: Anat, A, Duke, H. R., and Corey, A.T., Colorado State University, Hydrology Paper No. 7 (June 1965) "Steady Upward Flow from Water Tables"

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